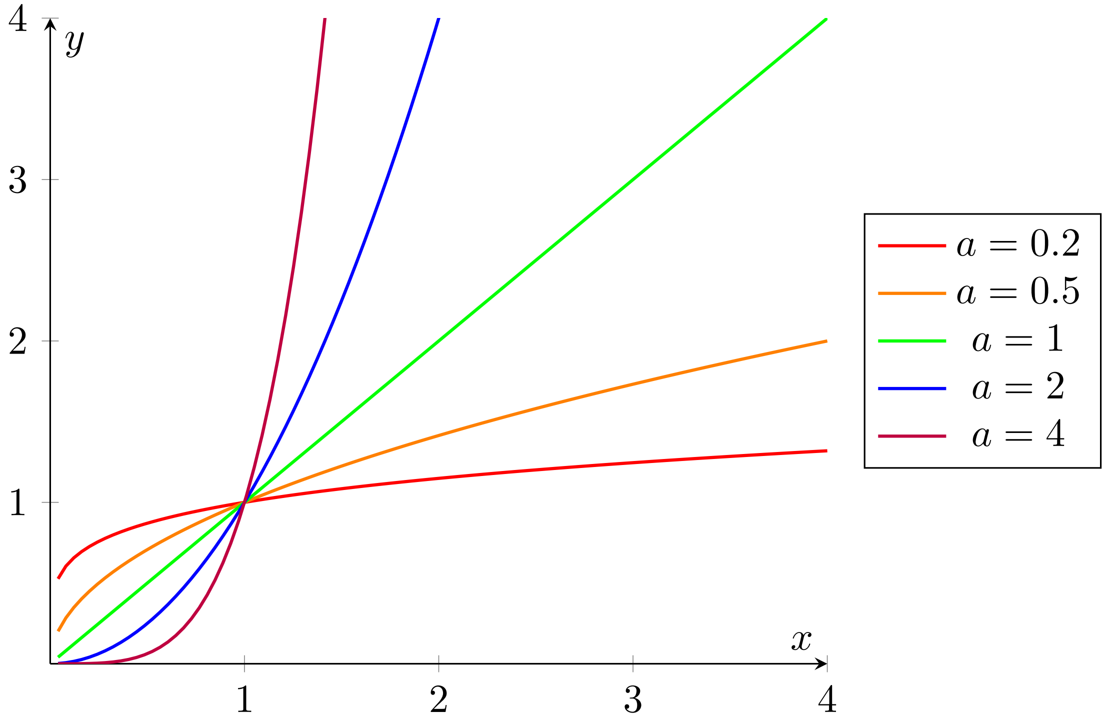
Fonctions Usuelles

Aubin SIONVILLE

MPI Clemenceau - 2021-2023

Fonctions puissances



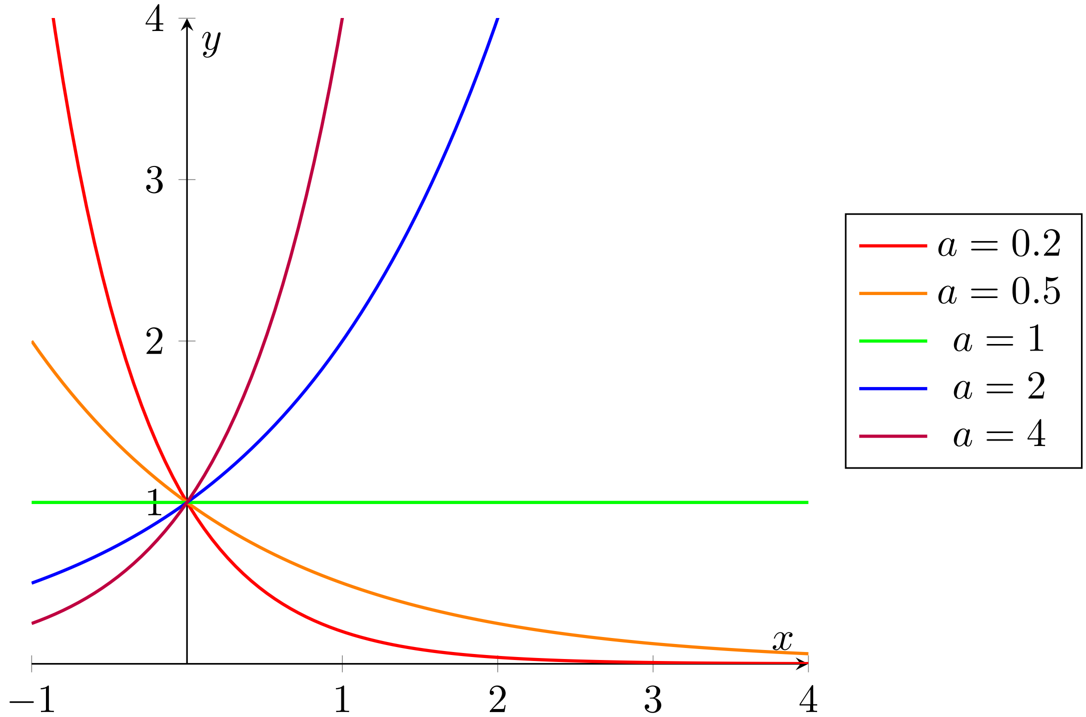
$$\forall a \in \mathbb{R}, \forall x > 0, x^a \triangleq \exp(a \ln x)$$

Hypothèse : $(a, b) \in (\mathbb{R}_+^*)^2$

$$\frac{\ln^a(x)}{x^b} \longrightarrow 0 \qquad \frac{\ln^a(x)}{\mathrm{e}^x} \longrightarrow 0$$

Exponentielle et logarithme de base a

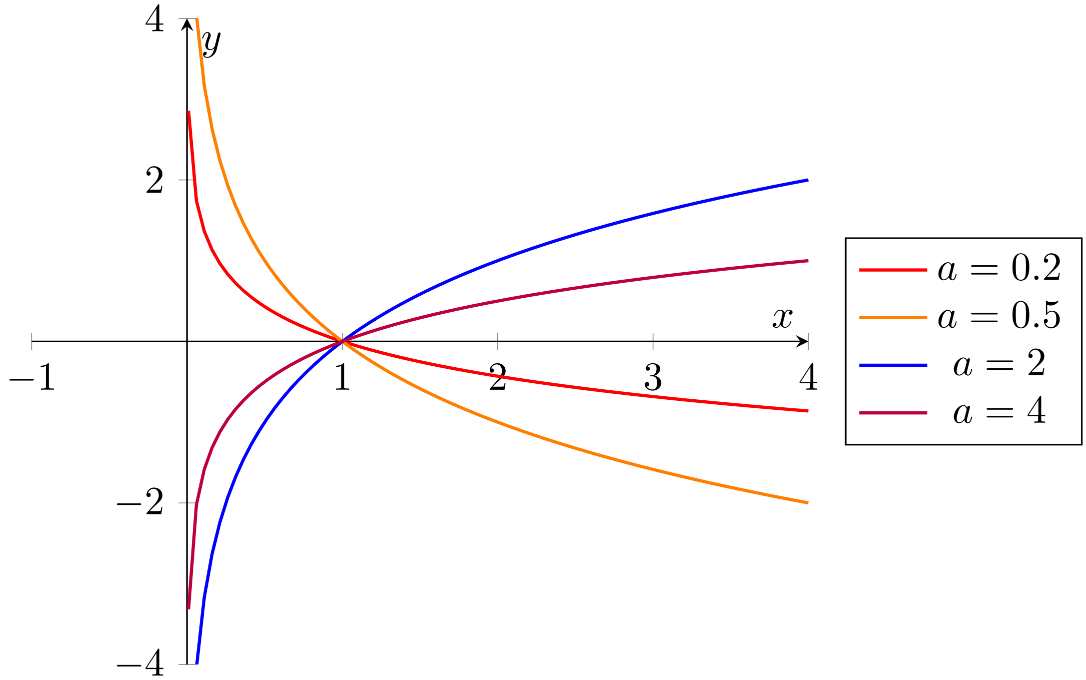
$$\boxed{\forall a > 0, \exp_a \triangleq \exp(x \ln a) = a^x}$$



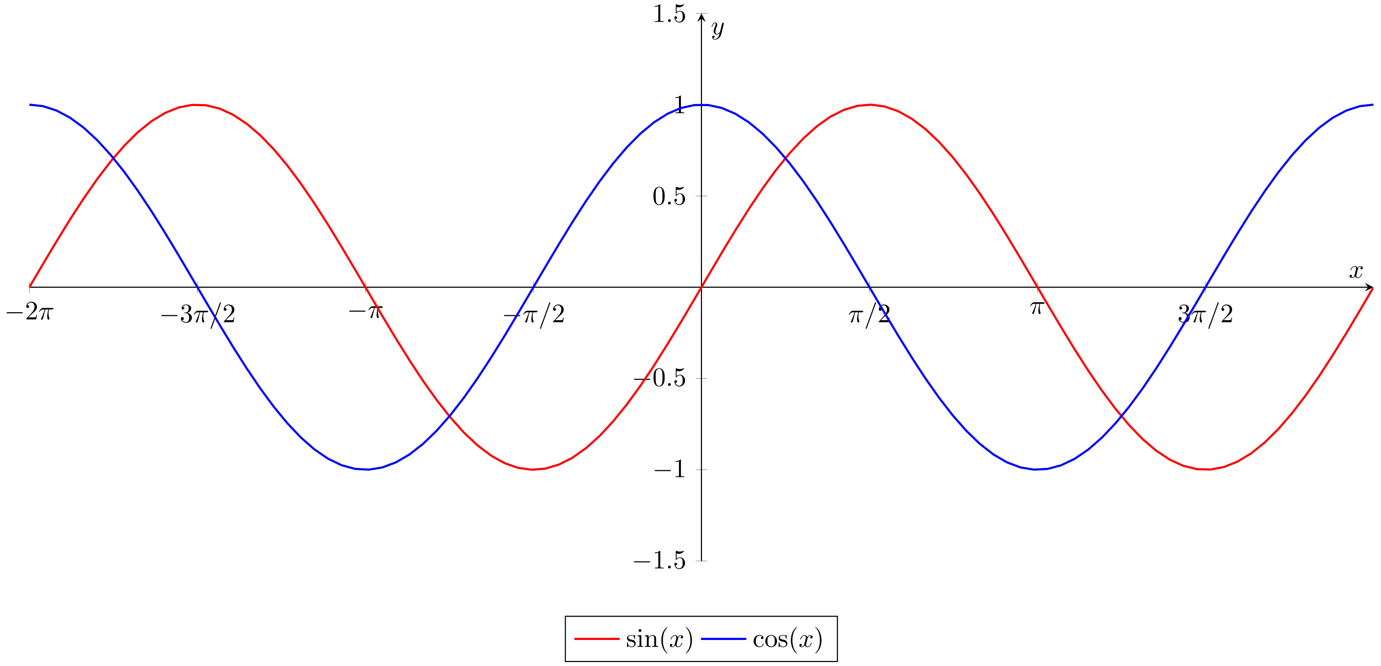
Pour $a \neq 1$: Réciproque : $\log_a$

Hypothèse : $a \in \mathbb{R}_+^* \setminus \{1\}$

$$\boxed{\forall x > 0, \log_a(x) \triangleq \frac{\ln(x)}{\ln(a)}}$$



Fonctions trigonométriques



$\boxed{\sin' = \cos}$

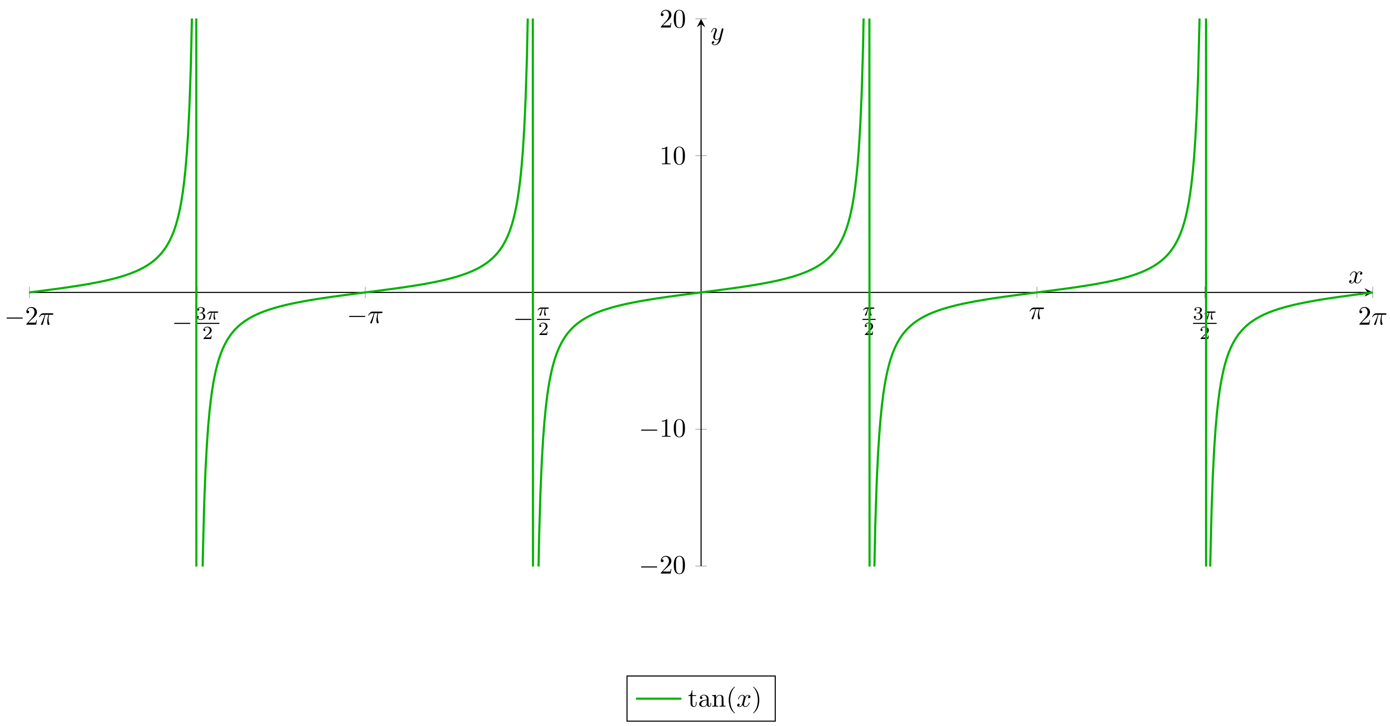
$\boxed{\cos' = -\sin}$

Pour $x \not\equiv \dfrac{\pi}{2}[\pi]$:

$$\boxed{\tan'(x) = 1 + \tan^2(x) = \frac{1}{\cos^2(x)}}$$

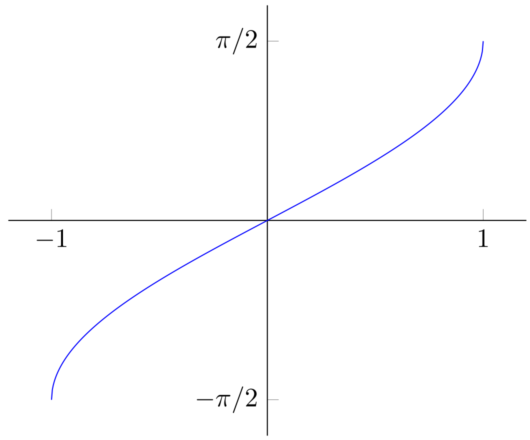
Pour $x \not\equiv 0[\pi]$:

$$\boxed{\text{cotan}'(x) = -1 - \text{cotan}^2(x) = \frac{-1}{\sin^2(x)}}$$



Fonctions trigonométriques réciproques

Arcsin



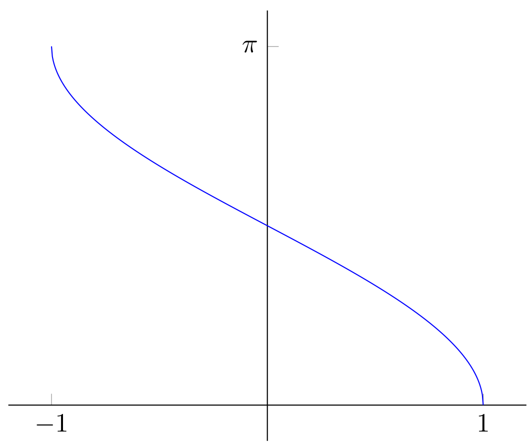
$$\text{Arcsin} \triangleq \text{ réciproque de } \begin{array}{cccc} f & : & \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] & \to & [-1,1] \\ & & x & \mapsto & \sin(x) \end{array}$$

Continue sur $[-1,1]$

Dérivable sur $]-1,1[$

$$\text{Arcsin}'(x) = \frac{1}{\sqrt{1-x^2}}$$

Arccos



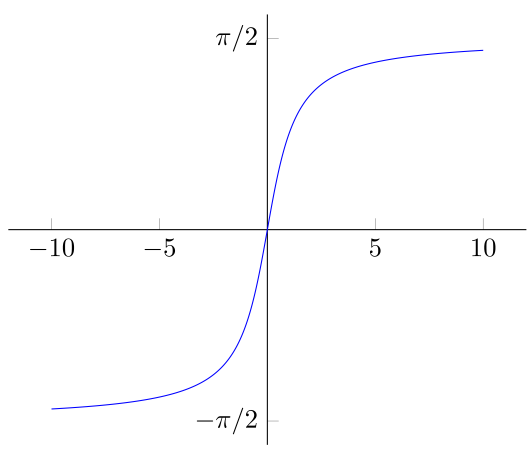
$$\text{Arccos} \triangleq \text{ réciproque de } \begin{array}{cccc} f & : & [0,\pi] & \to & [-1,1] \\ & & x & \mapsto & \cos(x) \end{array}$$

Continue sur $[-1,1]$

Dérivable sur $]-1,1[$

$$\text{Arccos}'(x) = \frac{-1}{\sqrt{1-x^2}}$$

Arctan



$$\text{Arctan} \triangleq \text{ réciproque de } \begin{array}{cccc} f & : & \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] & \to & \mathbb{R} \\ & & x & \mapsto & \tan(x) \end{array}$$

Impaire

Continue et dérivable sur $\mathbb{R}$

$$\text{Arctan}'(x) = \frac{1}{1+x^2}$$

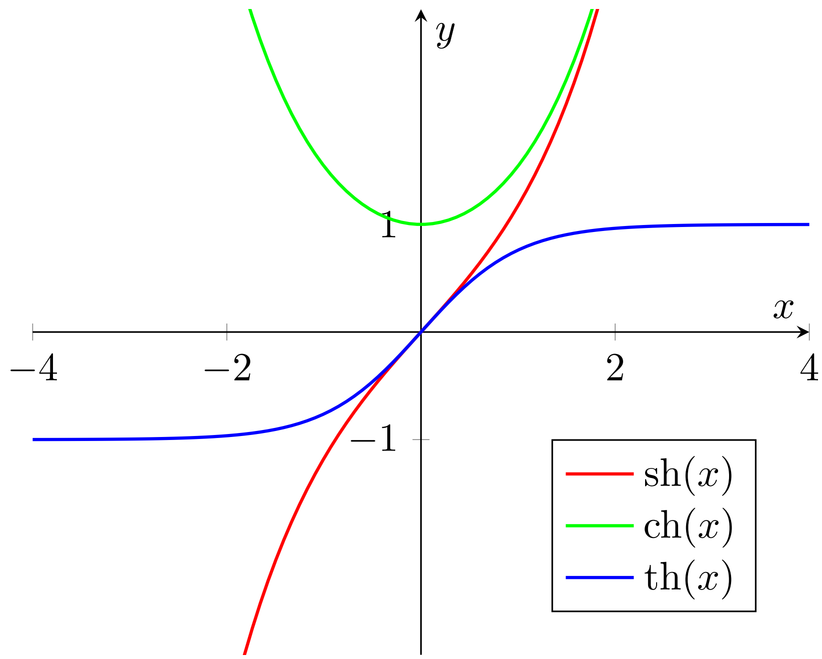
$$\text{Arctan}(x) + \text{Arctan}\left(\frac{1}{x}\right) = \begin{cases} \frac{\pi}{2} & \text{si } x > 0 \\ -\frac{\pi}{2} & \text{si } x < 0 \end{cases}$$

Trigonométrie hyperbolique

$\boxed{\mathrm{ch}(x) \triangleq \dfrac{e^x + e^{-x}}{2}}$

$\boxed{\mathrm{sh}(x) \triangleq \dfrac{e^x - e^{-x}}{2}}$

$\boxed{\mathrm{th}(x) \triangleq \dfrac{\mathrm{sh}(x)}{\mathrm{ch}(x)}}$



$\boxed{\mathrm{ch}^2(x) - \mathrm{sh}^2(x) = 1}$

$\boxed{\mathrm{ch}(a+b) = \mathrm{ch}(a)\mathrm{ch}(b) + \mathrm{sh}(a)\mathrm{sh}(b)}$

$\boxed{\mathrm{sh}(a+b) = \mathrm{sh}(a)\mathrm{ch}(b) + \mathrm{ch}(a)\mathrm{sh}(b)}$

$\boxed{\text{ch est paire}}$

$\boxed{\mathrm{ch}' = \mathrm{sh}}$

$\boxed{\text{sh est impaire}}$

$\boxed{\mathrm{sh}' = \mathrm{ch}}$

$\boxed{\text{th est impaire}}$

$\boxed{\mathrm{th}'(x) = 1 - \mathrm{th}^2(x) = \dfrac{1}{\mathrm{ch}^2(x)}}$