

Preuves

Aubin SIONVILLE

MPI Clemenceau - 2021-2023

Déduction naturelle intuitionniste

Règle d'Axiome : $\frac{}{\Gamma, \varphi \vdash \varphi} \text{Ax}$

Règle d'Affaissement : $\frac{\Gamma' \vdash \varphi}{\Gamma \vdash \varphi} \text{Aff} \quad (\Gamma' \subset \Gamma)$

\top	$\frac{}{\Gamma \vdash \top} \top\text{i}$	\emptyset
\perp	\emptyset	$\frac{\Gamma \vdash \perp}{\Gamma \vdash G} \perp\text{e}$
\neg	$\frac{\Gamma, G \vdash \perp}{\Gamma, G \neg G} \neg\text{i}$	$\frac{\Gamma \vdash G \quad \Gamma \vdash \neg G}{\Gamma \vdash \perp} \neg\text{e}$
\rightarrow	$\frac{\Gamma, G \vdash H}{\Gamma \vdash G \rightarrow H} \rightarrow\text{i}$	$\frac{\Gamma \vdash G \rightarrow H \quad \Gamma \vdash G}{\Gamma \vdash H} \rightarrow\text{e}$
\wedge	$\frac{\Gamma \vdash G \quad \Gamma \vdash H}{\Gamma \vdash G \wedge H} \wedge\text{i}$	$\frac{\Gamma \vdash G \wedge H}{\Gamma \vdash G} \wedge\text{e,g} \quad \frac{\Gamma \vdash G \wedge H}{\Gamma \vdash H} \wedge\text{e,d}$
\vee	$\frac{\Gamma \vdash G}{\Gamma \vdash G \vee H} \vee\text{i,g} \quad \frac{\Gamma \vdash H}{\Gamma \vdash G \vee H} \vee\text{i,d}$	$\frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash G \quad \Gamma, B \vdash G}{\Gamma \vdash G} \vee\text{e}$

Déduction naturelle classique

Tiers Exclu

Double Négation

Absurde

$\frac{}{\Gamma \vdash G \vee \neg G} \text{TE}$

$\frac{\Gamma \vdash \neg \neg G}{\Gamma \vdash G} \neg\neg\text{e}$

$\frac{\Gamma, \neg G \vdash \perp}{\Gamma \vdash G} \text{Abs}$

Premier ordre de la deduction naturelle

\forall	$\frac{\Gamma \vdash G}{\Gamma \vdash \forall x G} \forall\text{i}$ <small>Avec $x \notin FV(\Gamma)$</small>	$\frac{\Gamma, \vdash \forall x G}{\Gamma \vdash G[x \mapsto t]} \forall\text{e}$ <small>Avec $vars(t) \cap BV(G) = \emptyset$</small>
\exists	$\frac{\Gamma \vdash G[x \mapsto t]}{\Gamma \vdash \exists x G} \exists\text{i}$	$\frac{\Gamma \vdash \exists x H \quad \Gamma, H \vdash G}{\Gamma \vdash G} \exists\text{e}$ <small>Avec $x \notin FV(\Gamma) \cup FV(G)$</small>